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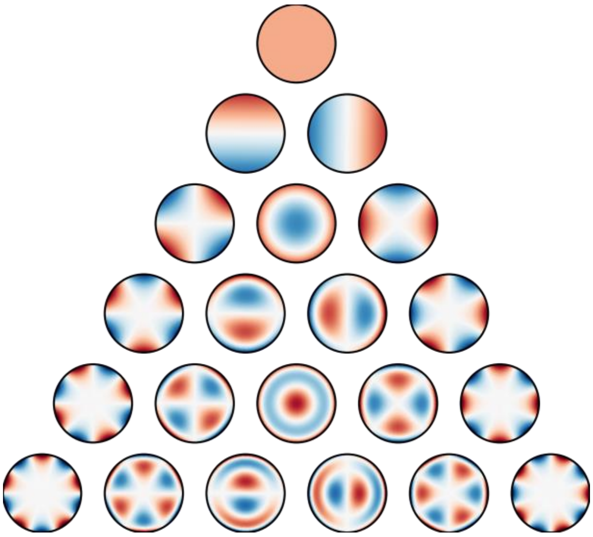

Calculation of beam parameters from local slopes

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APPLICATION NOTE

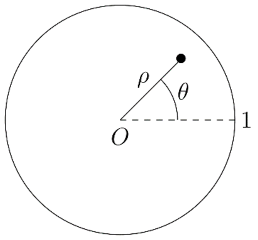
When a light wave passes through an optical system, we can consider a phase shift function φ corresponding to the difference between the theoretical perfect wavefront and the one carrying the optical defects of the system. φ is equal to zero if the system is perfect. In case of a circular pupil, these phase differences can be approximated by projecting the phase map onto the orthogonal basis of Zernike polynomials, the elements that describe wavefront aberrations.

1. Expression of the wavefront

A wavefront φ can be written as a sum of the first five Zernike polynomials and a residual wavefront φ_r containing all other aberrations:

$$\varphi(\rho,\theta) = Z_1\rho\cos(\theta) + Z_2\rho\sin(\theta) + Z_3\cdot(2\rho^2-1) + Z_4\rho^2\cos(2\theta) + Z_5\rho^2\sin(2\theta) + \varphi_r(\rho,\theta)$$

where (ρ,θ) is the polar coordinates on the circular **normalized** pupil (*i.e.* $\rho \in [0;1]$), and Z_i is the *i*-th Zernike coefficient.



• Coefficients Z_4 and Z_5 are corresponding to the astigmatism. By noting A as the **amplitude** of the astigmatism and θ_0 as its **angle**:

$$\left| \begin{array}{l} Z_4 = A\cos 2\theta_0 \\ Z_5 = A\sin 2\theta_0 \end{array} \right.$$

We can then rewrite the wavefront as

$$\boxed{\begin{aligned}\varphi(\rho,\theta) = Z_1\rho\cos(\theta) + Z_2\rho\sin(\theta) + Z_3\cdot(2\rho^2-1) \\ + A\rho^2\cos\big(2(\theta-\theta_0)\big) + \varphi_r(\rho,\theta)\end{aligned}}$$

2. Calculation of the radius of curvature of astigmatism along their axes

We can express the radius of curvature of the astigmatism along their axes (Ox and Oy in Figure 1).

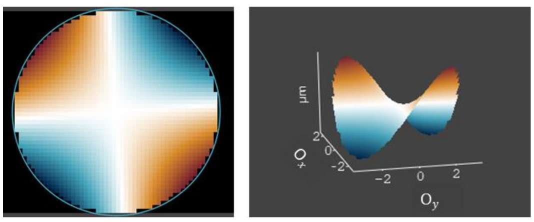


Figure 1. Representation of astigmatism on a circular pupil.

The wavelines along these axes become:

$$\begin{aligned}\varphi(\rho,\theta_0) &= Z_3(2\rho^2-1) + A\rho^2 \\ &= (\rho r)^2\left(\frac{2Z_3+A}{r^2}\right) - Z_3\end{aligned}$$

$$\begin{aligned}\varphi\left(\rho,\theta_0 + {}^{\pi}/_{2}\right) &= Z_3(2\rho^2-1) - A\rho^2 \\ &= (\rho r)^2\left(\frac{2Z_3-A}{r^2}\right) - Z_3\end{aligned}$$

• The coefficients in front of the terms in $(\rho r)^2$ are associated with the curvature $2\gamma = \frac{1}{2R_c}$. We now have:

$$\boxed{\begin{array}{c}\dfrac{1}{2R_{\theta_0}} = \dfrac{2Z_3+A}{r^2} \\[2ex] \dfrac{1}{2R_{\theta_0+\pi/2}} = \dfrac{2Z_3-A}{r^2}\end{array}}$$

3. Calculation of the radius of curvature of astigmatism along the axes of a wavefront sensor

When using a wavefront sensor to detect aberrations, , we are interested in expressing the radius of curvature of the astigmatism this time **along the main axes of an wavefront sensor itself**.

• Let's go back to the wavefront equation :

$$\varphi(\rho,\theta) = Z_3\cdot(2\rho^2-1) + A\rho^2\cos\big(2(\theta-\theta_0)\big) + \varphi_r(\rho,\theta)$$

We can find the expression of the waveline on the axes of the analyzer:

$$\left| \begin{array}{l} \varphi(\rho,0) \quad\; = (\rho r)^2\dfrac{2Z_3 + A\cos(2\theta_0)}{r^2} - Z_3 \\[2ex] \varphi\left(\rho,{}^{\pi}/_{2}\right) = (\rho r)^2\dfrac{2Z_3 - A\cos(2\theta_0)}{r^2} - Z_3 \end{array} \right.$$

This gives:

$$\boxed{\frac{1}{2R_0} = \frac{2Z_3 + A\cos 2\theta_0}{r^2}} \qquad \boxed{\frac{1}{2R_{\pi/2}} = \frac{2Z_3 - A\cos 2\theta_0}{r^2}}$$

4. Determination of parameters using local slopes

We have the radius of curvature as a function of the astigmatism parameters. It remains for us to know how to **obtain these parameters** in the particular case of **our wavefront sensor**.

- Written in the **Cartesian coordinates** ($x = r\rho\cos(\theta)$ and $y = r\rho\sin(\theta)$), we have:

$$\varphi(x,y) = \frac{Z_1}{r}x + \frac{Z_2}{r}y + \frac{2Z_3}{r^2}(x^2+y^2) + \frac{A}{r^2}(x^2-y^2)\cos(2\theta_0) + \frac{2A}{r^2}xy\sin(2\theta_0) + \varphi_r(x,y)$$

We thus obtain analytically the following wavefront **gradient** or local slopes along x and y directions:

$$\frac{\partial\varphi}{\partial x}(x,y) = \frac{Z_1}{r} + \left(\frac{4Z_3}{r^2} + \frac{2A\cos 2\theta_0}{r^2}\right)x + \frac{2A\sin 2\theta_0}{r^2}y + \frac{\partial\varphi_r}{\partial x}(x,y)$$

$$\frac{\partial\varphi}{\partial y}(x,y) = \frac{Z_2}{r} + \frac{2A\sin 2\theta_0}{r^2}x + \left(\frac{4Z_3}{r^2} - \frac{2A\cos 2\theta_0}{r^2}\right)y + \frac{\partial\varphi_r}{\partial y}(x,y)$$

- On its side, the wavefront analyzer measures local slopes on a regular mesh or a microlens array. A single local slope is obtained from one microlens.

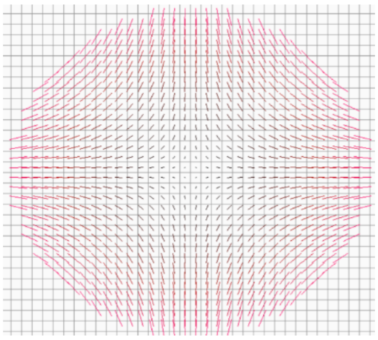


Figure 2. *Representation of slope map with astigmatism: Each point* (x_i, y_j) *is associated with a vector corresponding to the local slope.*

- We can separate the information of this vector map into two scalar maps along x and y direction, see the **Figure** 3.

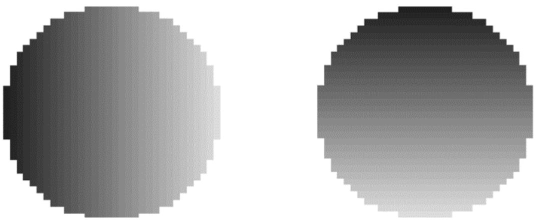
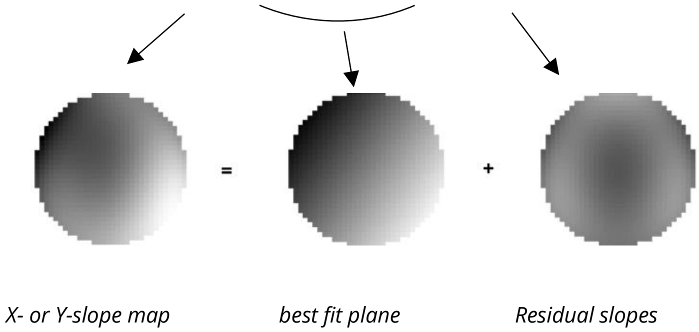


Figure 3. x-slope (left) and y-slope (right) maps.

We can then **numerically** find the **plane that fits a given wavefront surface** the best for each map and thus express the slope map in this way:

$$\begin{cases} s_x = a_x x_i + b_x y_j + c_x + s_{xres} \\ s_y = a_y x_i + b_y y_j + c_y + s_{yres} \end{cases}$$



⚠ The unit of coefficients is **slope per unit length**. The calculation of the nearest planes to the slope maps gives **slopes per sub-pupil**.

- Knowing the a_x, a_y, b_x and b_y coefficients, it is now a question of **extracting** θ_0 and A in order to fully determine the astigmatism, as well as the radius of curvature.

Note that in the expression of the gradient, the coefficients a_x and a_y are supposed to be equal. It is therefore recommended to **average** them in order to **minimize the impact of the measurement error** and to be consistent with the theory.

By **identifying** with the mathematical gradient expression we now have:

$$b_x^{'} = a_y^{'} = \frac{b_x + a_y}{2} = \frac{2A\sin(2\theta_0)}{r^2}$$

$$a_x = \frac{4Z_3}{r^2} + \frac{2A\cos(2\theta_0)}{r^2}$$

$$b_y = \frac{4Z_3}{r^2} - \frac{2A\cos(2\theta_0)}{r^2}$$

From this set of equations, we can deduce the **angle** of **astigmatism** θ_0:

$$\tan(2\theta_0) = \frac{b_x + a_y}{a_x - b_y}$$

Note If $a_x = b_y$, it is necessary to look at the value of $\frac{b_x + a_y}{2}$,

$$\frac{b_x+a_y}{2} = 0 \Rightarrow \boxed{A = 0} \quad \text{and} \quad \frac{b_x+a_y}{2} \neq 0 \Rightarrow \boxed{\theta_0 = \pi/4}$$

No astigmatism *Pure 45° astigmatism*

We deduce the **amplitude** of **astigmatism** A:

$$\boxed{A = \mathrm{sgn}(b_x + a_y) \cdot \frac{r^2}{2} \sqrt{\left(\frac{b_x + a_y}{2}\right)^2 + \left(\frac{a_x - b_y}{2}\right)^2}}$$

By identification, we can express the x and y **tilts** coefficients (resp. Z_1 and Z_2):

$$\boxed{Z_1 = r \cdot c_x} \quad \text{and} \quad \boxed{Z_2 = r \cdot c_y}$$

And finally the **Focus** coefficient Z_3 is

$$\boxed{Z_3 = r^2 \cdot \frac{a_x + b_y}{8}}$$